Optimal Control of a Family House Heating System

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Abstract—**Approximately 42% of total energy usage today is spent on the heating of the human environment. Energy consumption can be significantly reduced by proper design of the heating control system - performed studies suggest that in average about 12% reduction of energy usage can be achieved based on control system improvement. This paper describes an efficient control of a family house heating system in its very common configuration. We use model predictive control to fully take into account the prediction of disturbances (room temperatures), reference values of power per rooms and physical limitations of actuators. This will result in an optimal flow rate and temperature of medium required to meet all the demands. In this paper we will first present the model of a heating system, and from it derive model predictive control and verify it through simulation.**

I. Introduction

Each day we feel a lack of energy, therefore energy saving is a general social task. Correct automatic control of temperature and mass flow of the heating medium is the most effective way to save energy in heating systems. The heating system is designed for single-story house with two rooms and the attic [1]. For such a model in [2] model predictive control strategy has been derived. As the external conditions (temperature, insolation, etc.) can for the future be determined only in certain limits of uncertainty, the stochastic approach to the problem is chosen. The stochastic approach guarantees the meeting of all limitations only inside an interval of certainty. The stochastic MPC (SMPC) controller commands the heating power input for the rooms in the house so that the outdoor conditions are fully exploited in keeping the room temperatures within the comfort limits, and therefore minimum additional energy for heating is spent. In the mentioned work the nature and performance of the heating system was neglected and it was assumed that the heating input can change instantaneously. However, this assumption is not valid for real systems. The obtained required power input for heating per rooms is constant within one hour and will be a reference that must be achieved by optimal control of the heating system.

The goal of optimal control of the heating system is the minimization of energy consumption required to heat-up the heating medium and operate the pump, and also minimization of the deviation of heating powers brought into the room from the required values necessary to satisfy the comfort limits. Model predictive control fully takes into account the prediction of disturbances (room temperatures), reference values of powers per rooms and physical limitations of actuators. The model of the central heating system is highly nonlinear, and it must be iteratively linearized at each optimization step in order to implement the model predictive control. The result will be the optimum flow rate and temperature of the heating medium which will satisfy all demands and minimize the power consumption.

Section II describes the model of the heating system, section III deals with mathematical formulation of the optimization problem, and in section IV we verify the control strategy through simulations.

II. Modeling of the heating system

The heating system consists of two radiators, one for each room, piping, a variable speed circulation pump and a boiler with a thermostat. The heating system is designed for a single-story house with two rooms and the attic, but only the ground floor is heated. A diagram of the heating system is shown in Figure 1.

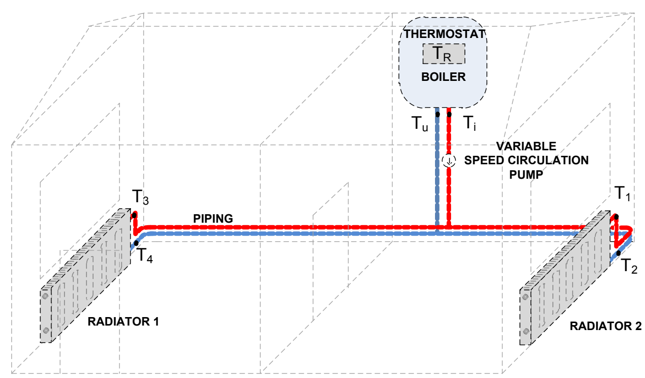

Fig. 1. Diagram of the heating system

A. Radiator

A radiator is part of a heating system which is used to transfer heat, mostly from hot water to the air. The working principle is based on the passage of a heating medium heated in a boiler through the radiator where the heat transmission medium heats up the radiator and loses heat. Radiators used to heat the ground floor are aluminum radiators composed of 25 segments. The number of segments was determined according to the recommendations of the radiator manufacturers. By installing a larger number of segments the initial costs are higher, but lower water temperature can be used for the same heat effect, thus the energy is saved and the regulation of room temperatures is faster.

The mathematical model is based on the assumption that the radiation flux of the aluminum radiator is negligible in relation to the convection flux. A mathematical description of the heat transfer inside radiator is obtained assuming that the water temperature in the radiator does not depend on spatial coordinates and is equal to the average temperature of the water:

$$m_v c_v \frac{dT_{iz}}{dt} = c_v \frac{Q}{2}(T_{ul} - T_{iz}) - \alpha_r A_r \left(\frac{T_{ul} + T_{iz}}{2} - T_z\right), \quad (1)$$

where T_{ul} is the temperature of the heating medium at the entrance of the radiator, T_{iz} is the temperature of the heating medium at the exit from the radiator, Q is the mass flow of the heating medium from the boiler, c_v is the specific heat capacity of the heating medium, α_r is the overall heat transfer coefficient form heating medium to air, A_r is the active surface area from which heat is transferred to the air, m_v is the mass of the heating medium in the radiator and T_z is the temperature of air in the room.

From the Figure 1 it is evident that the modeled heating system is a closed system so mass flow Q is the same throughout the entire system, whereas it is assumed that both branches of pipes have the same resistance, such that each radiator receives half of the heating medium mass flow.

B. Piping

Piping is a part of the heating system used to transfer the heat from the heat source to the heat exchangers using a suitable heating medium. The pipeline is modeled using four segments, a hot and a cold pipe for each room separately. The heat conservation equation for one segment is:

$$Q c_v T_{ulc} = Q c_v T_{izc} + 2\alpha_c A_c \left(\frac{T_{ulc} + T_{izc}}{2} - T_z\right), \quad (2)$$

where T_{ulc} and T_{ilc} are temperatures of the heating medium at the entrance and exit of the segment, α_c is the overall heat transfer coefficient form heating medium to air through a copper transmission surface and A_c is the active surface area from which heat is transferred to air.

C. Boiler

Hot water, i.e. the heating medium is produced in the boiler. Assuming that the temperature disposition in the boiler is homogeneous, the power of the boiler is:

$$\eta P_b = c_v Q (T_i - T_u), \quad (3)$$

where η is the usability factor, T_u is the temperature of the water returning from the system to the boiler, and T_i is the temperature of water at the exit from the boiler. To avoid nonlinearity which occurs in real systems when modeling a thermostat, it's assumed that the temperature at the exit from the boiler has PT_1 behavior in relation to reference temperature T_R,

$$T_b \frac{dT_i}{dt} + T_i = T_R, \quad (4)$$

where T_b is the time constant of the boiler. If we assume that the boiler is gas powered, the consumption of gas can be minimized directly by minimizing the power of the boiler.

D. Variable speed circulation pump

A circulation pump is a device which is used to transfer fluid (i.e. heating medium) from a lower to a higher level, i.e. from lower to higher pressure. The advantage of using the pump in heating systems is a faster circulation of the hot medium through the system and thus heating the space faster and achieving the desired comfort faster. The power of the pump is proportional to the flow to the third power:

$$P_p \sim Q^3. \quad (5)$$

In order to use quadratic solvers, at each step of the optimization the cubic power characteristic of the pump is approximated by a square characteristic around the predicted optimal mass flow along prediction horizon calculated in the previous step. As the mass flow of the heating medium is regulated through the optimal control of the variable speed circulation pump, the thermostat radiator valves are not necessary.

E. Overall model of the heating system

All the described elements of the heating system together form a closed system of a central heating. Inputs into the system are defined by the flow pump Q and a reference temperature of the water in the boiler T_R. Air temperatures of the east T_{ze} and west T_{zw} room are disturbances which act on the heating process. Temperatures at the exit from the radiators (T_2, T_4) of eastern and western room and output boiler temperature T_i are system states. Outputs of the system are the returning water temperature T_u and the power transmitted into the eastern P_e and the western P_w room. The described model of the central heating system is a highly nonlinear system and for the implementation of model predictive controller it must be iteratively linearized around the predicted optimal values calculated in the previous optimization step and the familiar amounts of disturbance along the prediction horizon. Nonlinear terms which describe the heating system can be found in [3]. Linearized system can be expressed in state space:

$$\begin{bmatrix} \Delta\dot{T}_i \\ \Delta\dot{T}_2 \\ \Delta\dot{T}_4 \end{bmatrix} = \begin{bmatrix} -1/T_b & 0 & 0 \\ K_{Ti} & K_T & 0 \\ K_{Ti} & 0 & K_T \end{bmatrix} \begin{bmatrix} \Delta T_i \\ \Delta T_2 \\ \Delta T_4 \end{bmatrix} + \begin{bmatrix} 1/T_b & 0 \\ 0 & K_{Qe} \\ 0 & K_{Qw} \end{bmatrix} \begin{bmatrix} \Delta T_R \\ \Delta Q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ K_{Tz} & 0 \\ 0 & K_{Tz} \end{bmatrix} \begin{bmatrix} \Delta T_{ze} \\ \Delta T_{zw} \end{bmatrix}. \quad (6)$$

Linearized output equations are:

$$\begin{bmatrix} \Delta T_u \\ \Delta P_e \\ \Delta P_w \end{bmatrix} = \begin{bmatrix} 0 & \Theta_T & \Theta_T \\ \Theta_{P,Ti} & \Theta_{P,T} & 0 \\ \Theta_{P,Ti} & 0 & \Theta_{P,T} \end{bmatrix} \begin{bmatrix} \Delta T_i \\ \Delta T_2 \\ \Delta T_4 \end{bmatrix} + \begin{bmatrix} 0 & \Theta_Q \\ 0 & \Theta_{P,Qe} \\ 0 & \Theta_{P,Qw} \end{bmatrix} \begin{bmatrix} \Delta T_R \\ \Delta Q \end{bmatrix} + \begin{bmatrix} \Theta_{Tz} & \Theta_{Tz} \\ \Theta_{P,Tz} & 0 \\ 0 & \Theta_{P,Tz} \end{bmatrix} \begin{bmatrix} \Delta T_{ze} \\ \Delta T_{zw} \end{bmatrix}. \tag{7}$$

The dependence of the system matrix coefficients on the stationary system state values can be found in [3]. Equations (6) and (7) fully describe the linearized heating system. The linearized system describes the given nonlinear system well only around a working point. Due to the high nonlinearity of the system in relation to the input mass flow, mass flow changes within the sampling time are limited to 20% of the stationary value.

III. Optimization

Use of automatic control in thermal processes can lead to significant energy savings. The heating system contains two actuators, which are managed with the purpose of reducing energy consumption. In [2] stochastic model predictive control is implemented on the same house model. The controller commands one hour constant heating power for rooms in the house so that the outdoor conditions are fully exploited in keeping the room temperatures within the comfort limits. The required heating power was obtained by neglecting the heating system performance and assuming that power of the actuator can be changed instantaneously. This assumption is not valid for real systems. So obtained required power will be a reference that must be achieved by optimal control of the heating system. The goal of optimal control of the heating system is the minimization of energy consumption required to heat up the medium and operate the pump, and the minimization of the deviation of power brought into the room from the required reference values necessary to satisfy the comfort limit.

$$\begin{aligned} \mathcal{J} &= \min_{\Delta T_R, \Delta Q} \Big[\int P_b + \int P_p + R \left(\int P_{ref,e} - P_e \right)^2 \\ &+ R \left(\int P_{ref,w} - P_w \right)^2 \Big], \end{aligned} \tag{8}$$

where R is the weighting matrix, through which the weight of minimization of integral square deviation is specified. The weighting matrix determines how important it is to satisfy the requirements for minimal quadratic deviation from the referent value of power in relation to the requirement for minimizing the power of the boiler and the pump. Model predictive control fully takes into account the prediction of disturbances (room temperatures), reference values of power per rooms and physical limitations of actuators. Stochastic model predictive control from [2], that produces references for heating power inputs for the rooms, takes into account the outside temperature influence as well as the influence of solar irradiance on the house outer envelope along the prediction horizon. Additionally on this level also the heat load from occupancy and other activities in the rooms could be taken into account.

Integrals of boiler and pump power and the exerted power error in each room are obtained by expressing the above powers dependence as a function of inputs, disturbances and initial state. Dependence of mentioned powers of disturbances and system inputs can be described using PT_1 and PT_2 transfer functions. For PT_1 and PT_2 transfer functions there are analytical expressions for the step response, with which the given power integrals can easily be solved. The exact expressions for integral of the boiler and the pump power, and the power submitted in each room can be found in [2].

By solving the integrals mentioned in the optimization criterion, the quadratic optimization criterion problem is obtained. The optimization criterion in form of a quadratic problem is:

$$\mathcal{J} = \min_{\mathbf{X}_k} \frac{1}{2} \mathbf{X}_k' \mathbf{H}_k \mathbf{X}_k + \mathbf{f}_k' \mathbf{X}_k, \tag{9}$$

where vector $\mathbf{X}_k$ contains the optimal input values along the prediction horizon:

$$\mathbf{X}_k = \begin{bmatrix} \Delta T_{R,k|k} \\ \Delta Q_{k|k} \\ \Delta T_{R,k+1|k} \\ \Delta Q_{k+1|k} \\ \vdots \\ \Delta T_{R,k+N-1|k} \\ \Delta Q_{k+N-1|k} \end{bmatrix}. \tag{10}$$

$T_{R,k+1|k}$ presents the predicted optimal value of the reference boiler temperature at time $(k+1)$ obtained on the basis of the data available at the time k (disturbance profile and reference power values along the prediction horizon). Matrices $\mathbf{H}_k$ and $\mathbf{f}_k$ are matrices which define the quadratic problem, where the $\mathbf{H}_k \in R^{2N \times 2N}$ and $\mathbf{f}_k \in R^{2N}$. Elements of these matrices were obtained by the linearization of the heating system around the predicted optimal input values along prediction horizon from the previous step and the familiar amounts of disturbance along the prediction horizon. Linearization is therefore performed iteratively at each step. Algorithm of iterative linearization can be described using the following steps:

- System initialization : $k = 1$, $\mathbf{U}_0 \in R^{2N}$;

1. Linearization of nonlinear system around the predicted optimal input values along the prediction horizon from the previous step $\mathbf{U}_{k-1}$ and the familiar amounts of disturbance along the prediction horizon $T_{D,k}$,

$$\mathbf{U}_{k-1} = \begin{bmatrix} T_{R,k-1|k-1} \\ Q_{k-1|k-1} \\ \vdots \\ T_{R,k+N-2|k-1} \\ Q_{k+N-2|k-1} \end{bmatrix}, \mathbf{T}_{D,k} = \begin{bmatrix} T_{d,k} \\ T_{d,k+1} \\ \vdots \\ T_{d,k+N-1} \end{bmatrix}, \tag{11}$$

   where $T_{d,k} = \begin{bmatrix} T_{ze,k} & T_{zw,k} \end{bmatrix}$;

2. Definition of the quadratic optimization problem in form (9) - determination of matrices $\mathbf{H}_k$ and $\mathbf{f}_k$;

3. Obtaining the vector $\mathbf{X}_k$ by solving a finite horizon quadratic optimal control problem using a developed code in Matlab;
4. Obtaining the optimal input values $\mathbf{U}_k$=$\mathbf{U}_{k-1} + \mathbf{X}_k$;
5. Applying the $T_{R,k|k}$ and $Q_{k|k}$ to the system;
6. Time update $k = k + 1$.
7. Back to step 1;

The exact expressions are given in [3].

Mass flow given by the circulation pump is restricted to the interval $0.01 - 1\ kg/s$, referent temperature of the boiler is restricted to the interval $0 - 100\ ^oC$ and boiler is $7\ kW$ sized. Additional restrictions are unnecessary because they follow directly from the given physical constraints of the system.

This optimization problem is solved by using Matlab Multi-parametric Toolbox [4] routines and appropriate quadratic programming solver like CPLEX [5].

IV. Results

The accuracy of the realized MPC regulator is tested by realizing a hierarchical heating process control system on two levels. On the higher level, using a SMPC regulator realized in [2], the optimal power input values per room necessary to maintain the desired temperature profile in the rooms are determined every hour through optimization. Optimal control of the heating system on a lower level gives the optimal amount of the mass flow and the necessary temperature of the heating medium, four times an hour, in order to spend as less energy as possible to get as little as possible deviation from the required reference power values generated by the higher control level. The central heating system generates the required power into the rooms by generating an optimal mass flow of the heating medium using a pump and heating the water in the boiler to a required optimal value.

The simulation was conducted during the first half of the year when the outside temperature values require the heating of interiors. The prediction horizon of both control levels is $N = 8$. The room temperatures must be kept within a limits of $20 - 25$ degrees with a probability of $95\ \%$. In line with the expectations and the given limitations, MPC on lower level calculates the optimal mass flow (Figure 5) and the referent boiler temperature (Figure 4) to satisfy the minimum deviation of the power transferred to the rooms from the demanded value generated by higher level, and at the same time satisfy the request for minimum boiler and pump power.

In Figures 4 and 5 the optimal input values into the heating system are shown. The heating system response to the calculated optimal inputs is shown in Figure 2 and the temperature response of western room to the input power generated by heating system is shown in Figure 3. Figure 6 shows an enlarged view of following the reference power during one day. It is obvious from Figure 6 that the ideal following the reference in the case of real actuators cannot be achieved. Optimal values of inputs to the heating system along the prediction horizon are found through optimization which minimize integrated squared difference between the actual and the reference value, taking into account that the user wants to

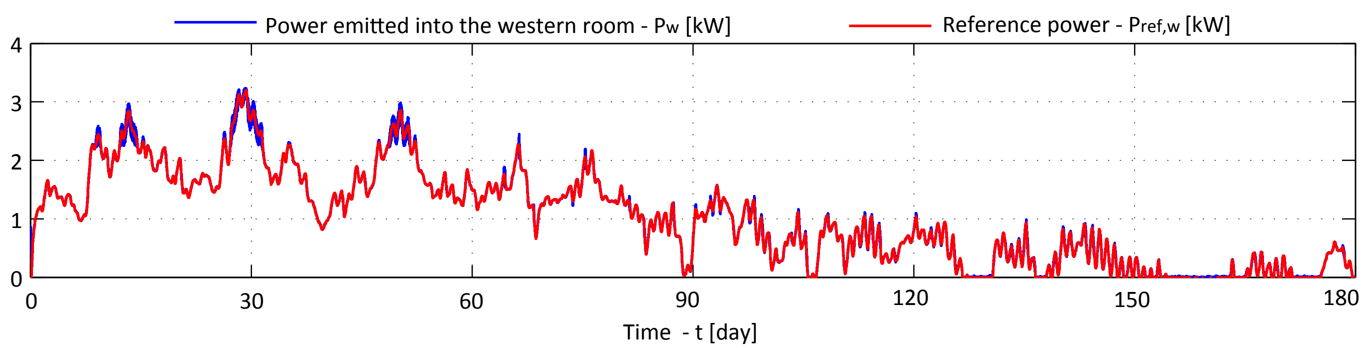

Fig. 2. Response of the heating system to the optimal inputs.

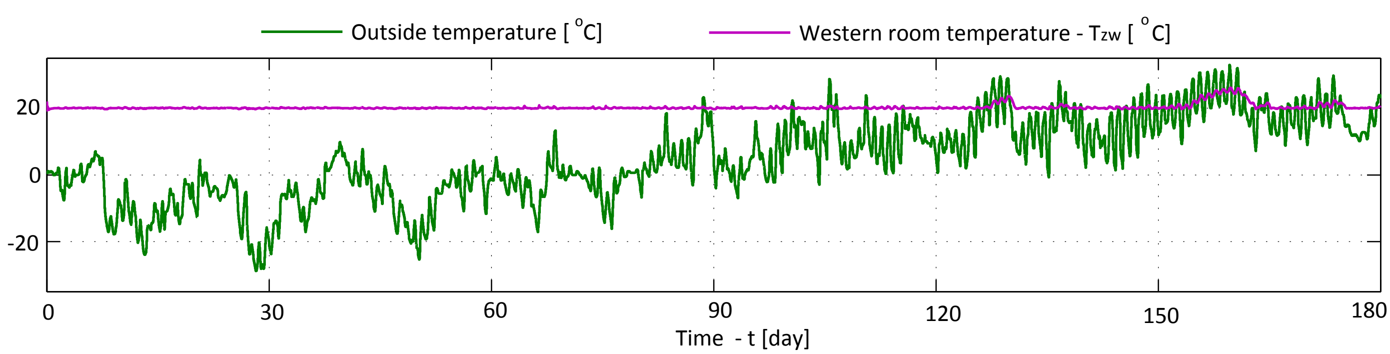

Fig. 3. Temperature response of the western room.

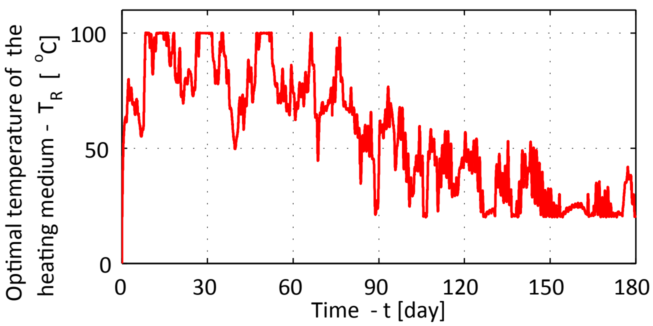

Fig. 4. Optimal temperature of the heating medium.

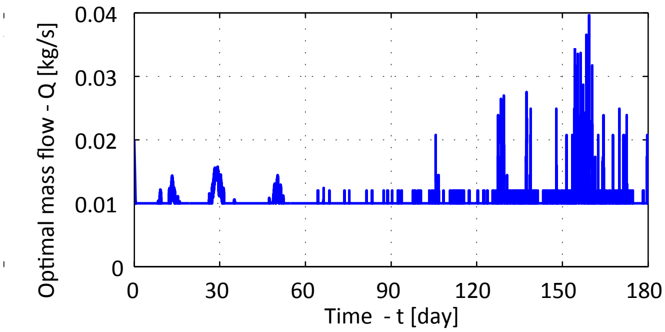

Fig. 5. Optimal mass flow.

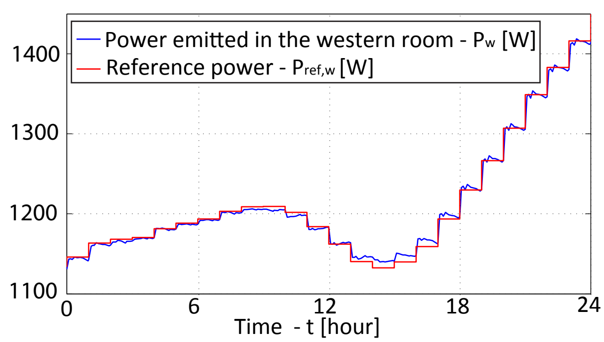

Fig. 6. Optimal temperature of the heating medium.

spend as little as possible for the required comfort (constant power within one hour ensures room temperatures maintained in the comfort interval), which means that the actuator power is as small as possible. The power submitted to the room can be increased by increasing the mass flow or by increasing the referent boiler temperature. The same applies to the demand for decreasing power submitted to the room.

From Figure 5 it can be seen that the temperature increase of the heating medium is an energetically cheaper solution to achieve the required power. For the previously stated reasons, with the temperatures of the heating medium inside the interval of $0 - 100\ ^oC$ the regulator keeps the mass flow on its minimal value (if there are no large leaps in the referent power requirements). With higher referent power values and the maximum temperature of the heating medium, the required referent power is gained by increasing the mass flow. With large leaps of the referent power value when it's necessary to achieve a large increase in the power submitted to the room in the minimum period of time (minimum deviation between the referent and the achieved power value), the necessary power value cannot be achieved fast enough only by heating up the heating medium, but it is necessary to increase the mass flow.

Low referent power values $(0 - 0.5kW)$ can be accomplished with lower temperatures of the heating medium. A higher level, aimed at minimizing the consumption of the actuators, generates low referent power values in case when the outdoor temperature is close to the user's defined comfort interval. The most efficient way of generating such power values is with the temperature of the heating medium close to the given comfort interval and the increase in mass flow (Figure 5 150th-180th day). There is no great decrease in the heating medium temperature in passing through the central heating system due to indoor temperatures within the given comfort interval, outdoor temperature close to the given interval and a high mass flow, so the power used by the boiler is minimal.

In spite of the stochastic behavior of the disturbance that affects the process [2] and the realization of the heating system which cannot follow the referent values ideally, we have managed to achieve a satisfactory behavior of the system due to the fact that the indoor temperature is within the interval set by the user (Figure 3).

Depending on the desired optimization effect, different reactions of the systems are accomplished by assigning different weights (unit energy costs) to the pump power and boiler power and changing the weighting coefficient R. In the conducted simulations where R=10, the emphasis was on minimizing the integral square deviations of powers.

V. Conclusion

In this paper we have modeled a heating system which consists of a variable speed circulation pump, a pipeline, a boiler and two radiators. The heating system is modeled for a house that has already been used to construct SMPC. The SMPC provides optimal input power demands which maintain indoor temperatures within the comfort interval set by the user. SMPC takes into account system dynamics, initial conditions, limitations and prediction of the disturbances along the prediction horizon and their uncertainty leaving enough space for preventing the disturbances in the following step from putting the system out of its limitations. Demanded optimal input power ensures maximal exploitation of natural resources (insolation, outside temperature, etc.) that we have at a certain time of the day. The problem is that the demanded optimal input power is constant within one hour, and obtained under the assumption of instantaneous changes of input power. Constant committed power and sudden steps are not achievable in a real system. In order to bring the optimal control of the temperature process as close as possible to the real system, hierarchical control has been realized. MPC of the heating system is used in order to achieve minimum possible deviation form the required amount of exerted heating power, and at the same time to minimize the energy consumed by boiler and circulation pump. Hierarchical control ensures a minimal deviation from the required referent power values with minimal energy expenditure and maintaining the temperature within the interval set by the user. Despite the high sensitivity of the central heating model to changes in input flow, because of high nonlinearity, the algorithm synthesized over the heating system ensures minimal deviation between the actual and the reference power at minimum cost of the consumed energy and room temperatures in the desired range.

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