Model Predictive Control of a Heating, Ventilation and Air Conditioning System

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Abstract - The largest energy consuming subsystem in a building is heating, ventilation and air conditioning (HVAC) system. It is thus essential to optimize the behavior of that subsystem in order to achieve energy savings. In this work focus is on efficient energy usage in a sample zone system consisting of two coupled zones. Ventilation, heating and cooling are done by air handling unit (AHU) and variable air volume (VAV) boxes. Optimization of the system means finding optimal control sequences for AHU and VAV boxes. The model of the coupled zones is linear, while the models of the AHU and VAV boxes are nonlinear. In order to find a suboptimal solution, which will keep zone temperature in given boundaries, while handling ventilation requirements, linear programming and linear zone system model are used to find appropriate cooling/heating powers, and then those powers are used to obtain control sequence for AHU and VAV boxes.

I. INTRODUCTION

There is a trend of reducing energy consumption in buildings today. One reason for that is economic, reducing energy expenses, and the other is environmental, since reducing energy consumption reduces CO_2 emissions. Building energy dissipation can be reduced in various ways which can be more or less invasive. Example of invasive solution is additional insulation of the house, while noninvasive is, for example, better control of heating and cooling. The cooling and heating system in this example consists of centralized AHU and localized VAV box. By implementing model predictive control (MPC) for this system significant energy savings can be made. MPC has significant advantages over traditional control like PID, since it utilizes information about future environment temperature and solar irradiation to optimize energy consumption.

In this paper, MPC control is implemented on a sample configuration of two coupled zones in which cooling and heating of zones is done by AHU and VAV boxes. The idea is to use state space model of the coupled zone system for synthesis of MPC that resides on linear programming. The MPC controller outputs the optimized heating/cooling powers for different zones. Next step is determining control sequence for AHU and VAV boxes which will accomplish these heating/cooling powers in the optimal way. Another task of the AHU is to supply outside air into the zones, so that the quality of zone air is preserved. In order to accomplish that ASHRAE Standard 62.1 [2] is implemented.

The idea behind is to make the control system development extremely scalable, since optimized heating/cooling powers for zones can be computed for a large number of zones if simple model of zones with lumped heating/cooling power is used. Accomplishing these heating/cooling powers via AHU / VAV setup is then posed as a static optimization problem.

Brief description of the considered zone setup is given in Section II. Synthesis of MPC by linear programming is described in Section III. Ventilation goals are given in Section IV. Section V. deals with the problem of obtaining control sequence for AHU and VAV boxes. Results are analyzed in Section VI.

II. THERMAL MODEL OF THE CONSIDERED ZONE SETUP

The zone setup for which control scheme is done is shown in Figure 1. There are two zones, with floor area of 25 meters squared and height of 3 meters. The complete list of materials used in the model of the zones can be found in [1]. The model is reduced to only 8 states in order to solve linear program faster.

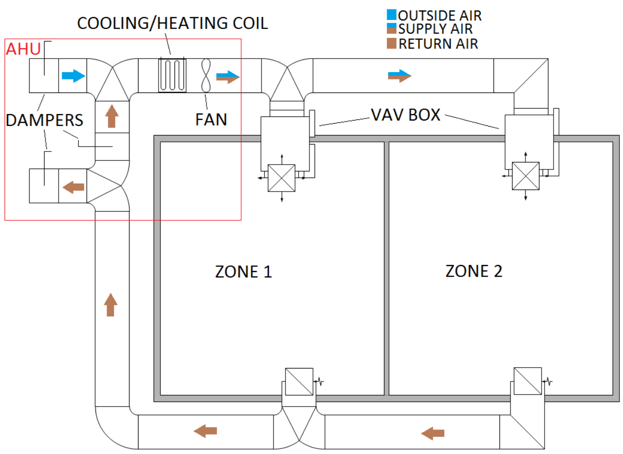

Figure 1 -Principle scheme of the zone setup.

III. SYNTHESIS OF MPC BASED ON LINEAR PROGRAMMING

Idea behind MPC is to find sequence of heating/cooling powers for zones which will keep zones temperatures in given boundaries on the prediction horizon, by taking into account current state of the model and all disturbances along the horizon. That way a sequence of input signals $U=(u_{t|t},u_{t+1|t},...,u_{t+N-1|t})$ is obtained. After that only the first input is applied, and the process is repeated in next step.

Model of the considered two-zone setup is continuous-time, so it must be discretized. Discretization is done by applying zero order hold with desired sampling time. The obtained discretized model can be described by:

$$x_{k+1}=A_d x_k + B_{du}u_k + B_{dd}d_k, \qquad (1)$$

where A_d, B_{du} and B_{dd} are discrete state matrix, input matrix and disturbance matrix respectively. Vector x_k, is state vector and contains temperatures of walls and zones, u_k is input vector and contains heating/cooling powers for zones, d_k is disturbance vector and contains outside temperatures and solar irradiations. System response on the horizon can be described by:

$$X=\alpha\cdot x_{t|t}+\beta\cdot U+\gamma\cdot D, \qquad (2)$$

where X is a stack of future states, U is a stack of future inputs and D is a stack of future disturbances:

$$\begin{aligned} X&=(x_{t+1|t},x_{t+2|t},...,x_{t+N|t});\\ U&=(u_{t|t},u_{t+1|t},...,u_{t+N-1|t});\\ D&=(d_{t|t},d_{t+1|t},...,d_{t+N-1|t}).\end{aligned} \qquad (3)$$

Matrices α, β and γ are:

$$\alpha=\begin{bmatrix}A_d\\ A_d^2\\ \vdots\\ A_d^N\end{bmatrix}$$

$$\beta=\begin{bmatrix}B_{du} & 0 & 0 & ... & 0\\ A_dB_{du} & B_{du} & 0 & ... & 0\\ A_d^2B_{du} & AB_{du} & B_{du} & ... & 0\\ \vdots & \vdots & \vdots & \ddots & 0\\ A_d^{N-1}B_{du} & A_d^{N-2}B_{du} & ... & A_dB_{du} & B_{du}\end{bmatrix} \qquad (4)$$

$$\gamma=\begin{bmatrix}B_{dd} & 0 & 0 & ... & 0\\ A_dB_{dd} & B_{dd} & 0 & ... & 0\\ A^2B_{dd} & A_dB_{dd} & B_{dd} & ... & 0\\ \vdots & \vdots & \vdots & \ddots & 0\\ A_d^{N-1}B_{dd} & A_d^{N-2}B_{dd} & ... & A_dB_{dd} & B_{dd}\end{bmatrix}$$

There are two limitations for this system. One is the requirement to keep temperatures of zones in given boundaries, and the other one limits heating and cooling power. These limitations can be described by:

$$\begin{aligned}\mathbb{X}^N&=\{X:\varepsilon_xX\le\xi_x\}\\ \mathbb{U}^N&=\{U:\varepsilon_uU\le\xi_u\}.\end{aligned} \qquad (5)$$

Matrices ε_x, ε_u, ξ_x, ξ_u are defined according to limitations. Keeping zones temperatures in given boundaries means meeting these constraints:

$$\begin{aligned}T_{\min}&\le T_1\le T_{\max}\\ T_{\min}&\le T_2\le T_{\max},\end{aligned} \qquad (6)$$

where T_1 and T_2 are zone temperatures while $T_{\min}$ and $T_{\max}$ are lowest and highest zone temperature allowed. The heating and cooling power constraints are:

$$\begin{aligned}U_{\min}&\le U_1\le U_{\max}\\ U_{\min}&\le U_2\le U_{\max},\end{aligned} \qquad (7)$$

where U_1 and U_2 are power inputs into zone one and two, while $U_{\min}$ and $U_{\max}$ are minimum allowed power and maximum allowed power respectively.

The goal of MPC is to minimize input power while maintaining zone temperature:

$$\begin{aligned}&\min_u \|U\|_1=\min_u\sum_{k=0}^{N-1}|u_k|,\\ &\begin{pmatrix}\varepsilon_x\beta\\ \varepsilon_u\end{pmatrix}U\le\begin{pmatrix}\xi_x-\varepsilon_x\alpha x_{t|t}-\varepsilon_x\gamma D\\ \xi_u\end{pmatrix}.\end{aligned} \qquad (8)$$

In order to define this as a linear program, absolute value nonlinearity must be resolved. That can be done by introducing slack variables Z, which meet the requirement $u_i\le z_i, -u_i\le z_i, i=1...N$, and minimize these variables. Now we can write our linear program like this:

$$\begin{aligned}&\min_{U,Z}\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}U\\ Z\end{bmatrix}\\ &\begin{pmatrix}\varepsilon_x\beta\\ \varepsilon_u\end{pmatrix}U\le\begin{pmatrix}\xi_x-\varepsilon_x\alpha x_{t|t}-\varepsilon_x\gamma D\\ \xi_u\end{pmatrix}\\ &-Z\le U\le Z.\end{aligned} \qquad (9)$$

This kind of problem can be solved using available linear programming solvers. Considered two-zone setup model response to optimal powers can be seen in Figure 2. The zone temperatures are kept between 20°C and 25°C. More on the topic of obtaining optimal cooling/heating powers can be found in [3].

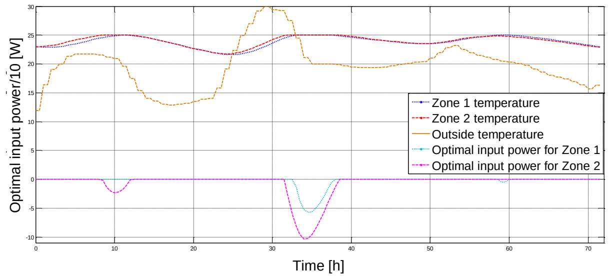

Figure 2 - Optimal input power response.

IV. RECIRCULATING VENTILATION SYSTEM AND ASHRAE STANDARD 62.1

With linear programming input cooling/heating powers which will be used in later control are obtained. Besides desired cooling/heating powers, ventilation criterions must be met. In order to preserve air quality in the zones ASHRAE Standard 62.1 [2] is applied.

The system that needs to be controlled consists of centralized AHU and localized VAV box. AHU's task is to mix return air and outside air, preheat or precool it. Mixing of the air is done by controlling air dampers. AHU also has supply fan, used to generate given airflow through supply duct. Every zone has its own VAV box, which is used to control air flow into the zone, and to heat up the air if necessary. The system control signal is $\{Q_1, Q_2, T_s, T_{s1}, T_{s2}, \rho\}$, where Q_1 is airflow to zone 1, Q_2 is airflow to zone 2, T_s is temperature of supply air from AHU to zones, T_{s1} and T_{s2} are temperatures of supply air for each zone from their VAV boxes and ρ is fraction of return air mixed into supply air. In order to simplify the model we assume that return airflow and supply airflow are equal, which means that input and output airflows for each zone are equal.

In order to implement ASHRAE Standard 62.1, minimal outside air intake flow, V_{ot}, must be determined. Firstly, breathing zone outdoor airflow V_{bzi} must be determined for each zone:

$$V_{bzi} = R_p \cdot P_{zi} + R_a \cdot A_z \tag{10}$$

- R_p - outside airflow rate required per person, in this paper it is set to $0.0025\left[\frac{m^3}{s}\right]$;
- P_{zi} - zone population, it is assumed that zone population is known through whole horizon;
- R_a - outdoor airflow rate required per unit area, in this work it is set to $0.0003\left[\frac{m^3}{m^2 s}\right]$;
- A_z - zone floor area, in this work it is $25\left[m^2\right]$.

Next step is determining zone outdoor airflow V_{ozi}:

$$V_{ozi} = \frac{V_{bzi}}{E_z}. \tag{11}$$

Coefficient E_z is called zone air distribution effectiveness, in this work it is set to 1, since it is assumed that there is ceiling supply air and floor return in each zone.

After that uncorrected outdoor air intake, V_{ou}, must be determined:

$$V_{ou} = D\sum_{i}^{N_z}(R_p \cdot P_z) + \sum_{i}^{N_z}(R_a \cdot A_z). \tag{12}$$

The occupancy diversity, D is set to 1, since it is assumed that population of the zone is known.

Last step is determining V_{ot}:

$$V_{ot} = \frac{V_{ou}}{E_v}. \tag{13}$$

Method for determining system ventilation efficiency, E_v, and a more detailed procedure for all other coefficients, can be found in [2].

Now all necessary requirements to proceed to final step are met, required heating/cooling powers to keep temperatures of the zones in given boundaries are obtained, and minimal outside air intake in order to comply with ASHRAE Standard 62.1 is also obtained, which means that air quality in zones is preserved.

V. FINDING CONTROL SEQUENCE WITH RESPECT TO COOLING/HEATING POWERS AND VENTILATION REQUIREMENTS

After required heating/cooling powers for zones and minimal outdoor air intake are determined, control sequence for AHU and VAV boxes can be calculated. The goal is to find control sequence such that heating/cooling powers for zones generated by AHU and VAV boxes correspond to heating/cooling powers obtained through MPC, for each zone, and airflow requirements are met, while energy spent for heating or cooling and recirculating air is minimized.

Power that AHU and VAV boxes are consuming can be put into this equation:

$$P_{total} = |P_{phc}| + P_{vav1} + P_{vav2} + P_{fan}. \tag{14}$$

Power P_{total} is sum of all powers, P_{phc} is preheating or precooling power in the AHU, P_{vav1} and P_{vav2} are powers of heating air in VAV boxes going into zone 1 and zone 2 respectively and P_{fan} is power used by fan to maintain required flow.

Preheating/precooling power is equal to:

$$P_{phc} = C_{air} Q_s \left(T_s - T_{mix}\right)$$
$$P_{phc} = C_{air} \left(Q_1 + Q_2\right)\left(T_s - \frac{\rho\left(Q_1 T_1 + Q_2 T_2\right) + (1-\rho)\left(Q_1 + Q_2\right) T_o}{\left(Q_1 + Q_2\right)}\right) \quad (15)$$

Flow Q_s is supply flow and it is equal to return flow, T_o is outside temperature, T_{mix} is temperature of mixed air and C_{air} is thermal capacity of air set to $1206\left[\frac{J}{m^3 K}\right]$.
Heating powers at VAV boxes are equal to:

$$\begin{aligned} P_{vav1} &= C_{air} Q_1 (T_{s1} - T_s); \\ P_{vav2} &= C_{air} Q_2 (T_{s2} - T_s). \end{aligned} \quad (16)$$

Temperatures T_{s1} and T_{s2} are temperatures of zone supply air for zone 1 and zone 2 respectively.

Power of the supply fan is approximated by:

$$P_{fan} = SFP \cdot \left(Q_1 + Q_2\right). \quad (17)$$

Coefficient SFP is specific fan power. In this paper it is set to $2\left[\frac{kW}{m^3 s}\right]$. More about SFP can be found in [4].
The goal is to minimize these powers with respect to constraints. Problem can be written like this:

$$\min_x f\left(x\right) such \quad that \begin{cases} c\left(x\right) \le 0 \\ ceq\left(x\right) = 0 \\ A \cdot x \le b \\ lb \le x \le ub. \end{cases} \quad (18)$$

Function $f\left(x\right)$ is the function that needs to be minimized, in this case:

$$f\left(x\right) = \left|P_{phc}\right| + P_{vav1} + P_{vav2} + P_{flow}, \quad (19)$$

because total power must be minimized. Argument of the function, vector x, is:

$$x = \left[Q_1 \quad Q_2 \quad T_s \quad T_{s1} \quad T_{s2} \quad \rho\right]^T. \quad (20)$$

Function $c(x)$ is a nonlinear constraint which sets minimal outdoor air intake required:

$$c(x) = \begin{cases} \frac{V_{oz1}}{Ev} - Q_1\left(1-\rho\right) \\ \frac{V_{oz2}}{Ev} - Q_2\left(1-\rho\right). \end{cases} \quad (21)$$

Airflow V_{oz1} and V_{oz2} are minimal required air intake flows for each zone. This way ASHRAE standard is applied.
In order to keep temperature in given boundaries, heating/cooling powers input for each zone must be equal to heating/cooling power input obtained by MPC. That is ensured with nonlinear function $ceq(x)$:

$$ceq(x) = \begin{cases} C_{air} Q_1\left(T_{s1} - T_1\right) - P_1 \\ C_{air} Q_2\left(T_{s2} - T_2\right) - P_2. \end{cases} \quad (22)$$

Matrix A, and vector b are linear constraints used to ensure that VAV box can only heat the air going into the zone. That can be done by setting difference between AHU supply air temperature, T_s and zone air supply temperatures, T_{s1} and T_{s2} less than or equal to zero. Matrix A, and vector b are:

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}; \\ b &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (23)$$

Vectors lb and ub are lower and upper bounds for all our variables:

$$\begin{aligned} lb &= \left[0 \quad 0 \quad 283 \quad 283 \quad 283 \quad 0\right]; \\ ub &= \left[0.5 \quad 0.5 \quad 318 \quad 318 \quad 318 \quad 1\right]. \end{aligned} \quad (24)$$

This kind of problem can be solved using *fmnicon* function in Matlab. The results are given in next section.

VI. RESULTS

In this chapter system behavior in couple of scenarios is analyzed.

A. Heating of the zones

If temperature outside is low heating/cooling powers will be applied to zones which will keep zones temperatures on the lower zone temperature boundary, in other words, zones will be only heated. Response of temperatures in that case can be seen in Figure 3. Temperatures which are result of applying optimal heating powers obtained by MPC, and response of temperatures of the system are the same. That means that heating/cooling powers for zones are well approximated using *fmincon* function.

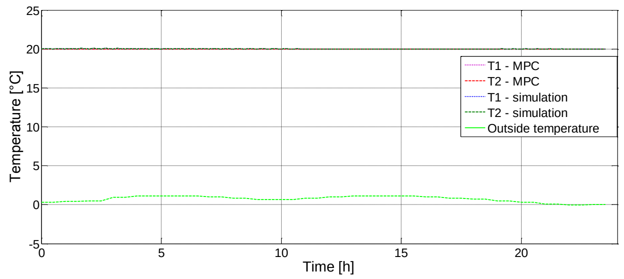

Figure 3 - Temperature response.

In Figure 4 minimal outside air intake for AHU, and zone occupancies are given. In a situation where outside air temperature is outside of the zone temperature boundaries, outdoor air intake should be minimal. It can be seen that this is true in this situation, since minimal air intake required by standard, and air intake that was obtained by MPC are the same.

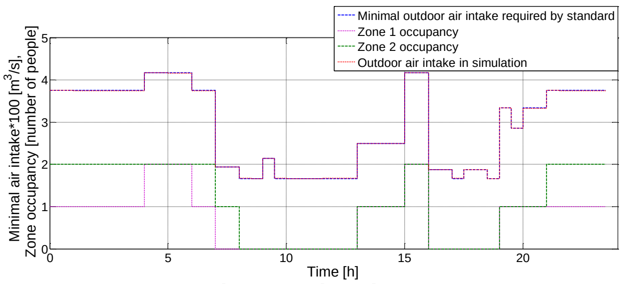

Figure 4 - Outdoor air intake and zone occupancy.

In Figure 5 total power used in system and obtained by MPC are given. Return air ratio $\rho \cdot 1000$ is also shown. It is obvious that AHU utilizes return air to minimize energy consumption. It can be seen that AHU consumes more power than obtained by MPC. That is because additional power is needed to heat up outside air coming into the system and to power the fan.

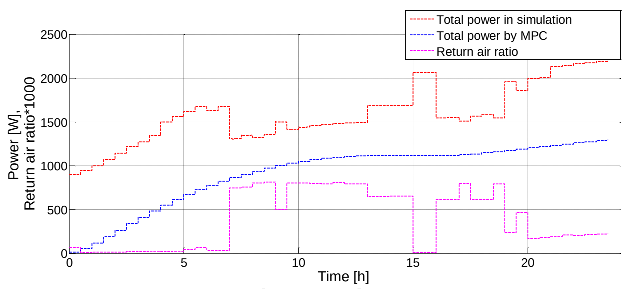

Figure 5 - Total power and return air ratio.

B. *Cooling the zones*

Cooling of the zones is the same as heating, if the outside air is above upper zone temperature boundary. But when the outside air temperature is inside given zone temperature boundaries, AHU should be able to utilize that to minimize energy consumption.

In Figure 6 temperature response is given. As before, zone temperatures are inside boundaries. All the times when outside temperature is above the required temperature interval for the zones and zone temperatures are on the upper boundaries, it can be concluded that exertion of cooling power to zones is needed to keep the zone temperature inside temperature boundaries.

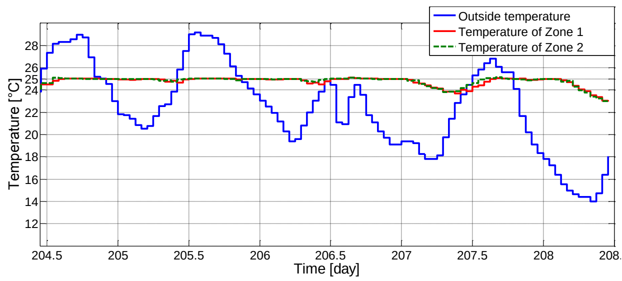

Figure 6 - Temperature response.

In Figure 7 outdoor air intake and zone occupancies are given. It can be seen that AHU takes in more outside air than required by standard, which means that AHU is utilizing outside air in order to minimize energy consumption.

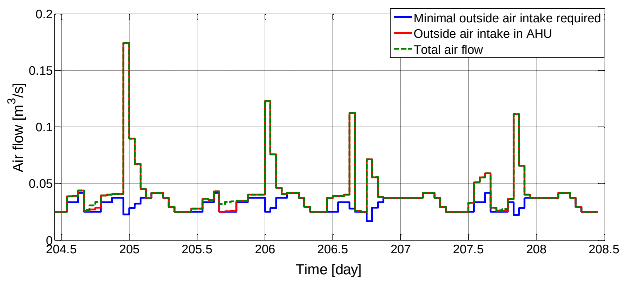

Figure 7 - Outdoor air intake.

In Figure 8 total power and return air ratio are given. It can be seen that AHU sometimes uses less power than demanded to be injected into zones by MPC. This confirms the assumption that AHU utilizes outside air in order to minimize energy consumption while delivering the right amount of cooling energy into the zones.

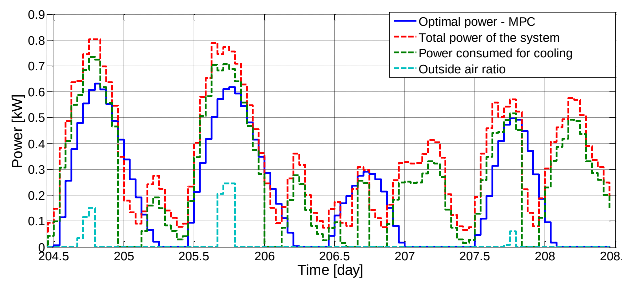

Figure 8 - Power response.

The case when the designed AHU/VAV control performs sub optimally is when cooling/heating powers obtained by MPC are zero and temperatures of the zones are somewhere inside zone temperature boundaries. Since AHU an VAV boxes produce equal cooling/heating powers as the ones obtained by MPC, and since AHU must take some air from the outside, it is possible that outside air will be heated (or cooled) to match that cooling/heating power. But since the temperature is not on the lower or upper boundary it is possible that more energy than needed will be consumed.

VII. Conclusion

It is clear that obtaining control sequence for AHU and VAV boxes in a manner described in this work could be used in practice. The nonlinearity and scalability problems are addressed by decomposing the control problem in two stages, computation of heating/cooling power exertions for zones and static optimization of HVAC system operation to adhere to the required power exertions to zones while ventilation constraint is also taken into account.

Acknowledgments

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References

[1] A. Starcic, Modeliranje i upravljanje toplinskim procesom u zgradi, Zagreb: Faculty of Electrical Engineering and Computing, 2010.

[2] ANSI/ASHRAE Standard 62.1-2007, Atlanta: ASHRAE, Inc., 2007.

[3] A. Martincevic, Modelsko prediktivno upravljanje toplinskim procesom u zgradi uz uzimanje u obzir nesigurnosti podataka iz vremenske prognoze, Zagreb: Faculty of Electrical Engineering and Computing, 2011.

[4] J. Railio i P. Mäkinen, »SPECIFIC FAN POWER – a tool for better performance of air handling,« , *Clima 2007 WellBeing Indoor*, Helsinki, 2007.